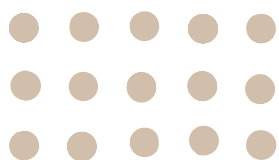"[N]umbers have quirks of structure that endow them with personalities" (Strogatz, 9).

Objectives :

① Discover "quirks" of numbers by working with them concretely, as "rock rectangles."

② Practice the process of mathematical investigation.

③ Work collaboratively to investigate a mathematical question.

Rectangles

Each number is represented by a group of "rocks."

15 rocks in a 5 by 3 rectangle

- For a given number of rocks, how many ways are there to arrange the rocks in a rectangular pattern?

- What does the number of rectangles tell us about the number of rocks?

For starters, look at six rocks.

- Six rocks → arrange in rectangles

   How many?   (More than one reasonable way to answer.)

(a) 4

(b) 2   b/c   Single row doesn't count
         & neither does single column

(c) 2   b/c   2×3   and   3×2   are   same
         & so are   1×6 and 6×1

(d) 1   b/c   single   row/col. don't count
         &   2×3 same as 3×2

In your group, ==choose a way to count== & be consistent.

(a) single row , single column both count
     m by n & n by m counted separately

(b) single row doesn't count; neither does single column
     m by n & n by m counted separately

(c) single row counts (& single column same as single row)
     m by n & n by m considered the same (so
     not counted separately)

(d) single row doesn't count; neither does single column;
     m by n & n by m considered the same (so
     not counted separately)

* For the sake of class discussion, make sure at least
   one group chooses to count as in (a) or (b).

What about 4 rocks?

(i) 3   (iii) 1

(ii) 2   (iv) 0

Note: In math, a square is considered a rectangle.

1. State what you decide about how to count!

2. For each number $1, 2, 3, \ldots, 12$, determine how many ways there are to arrange that many rocks into a rectangle. Record your work on the board as you go.

# rocks	rect. arrangements	# rect. arrangements
1		
2	(pictures or descriptions)	
3		(numbers)
4		
5		
6		
⋮		

3. Observe & Consider. What do you notice?
- Warm-up. (Closed-ended).

- Probe deeper, look for patterns.
  What does the # rectangles tell you about the # rocks?

# rectangles	# rocks

4&5 Predict, conjecture, & test. (Go up to 16 or 20.)
  – Confirm, reject, or revise conjectures.

6&7. Conclusions & explanations.
  – If you can explain why your conjecture is true, your conjecture becomes a conclusion.

8. Further questions.

  – Investigations often lead to new questions!